12.1 Exploring Solids

What you should learn

GOAL 1 Use properties of polyhedra.

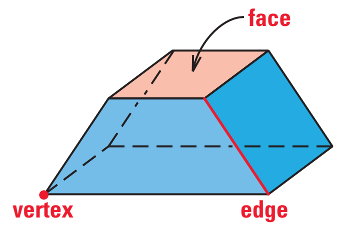
GOAL 2 Use Euler's Theorem in **real-life** situations, such as analyzing the molecular structure of salt in **Example 5**.

Why you should learn it

▼ You can use properties of polyhedra to classify various crystals, as in **Exs. 39–41**.

GOAL 1 USING PROPERTIES OF POLYHEDRA

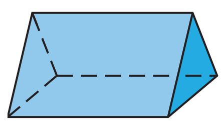
A **polyhedron** is a solid that is bounded by polygons, called **faces,** that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra*, or polyhedrons.



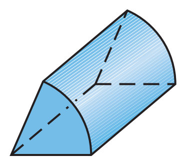
EXAMPLE 1 *Identifying Polyhedra*

Decide whether the solid is a polyhedron. If so, count the number of faces, vertices, and edges of the polyhedron.

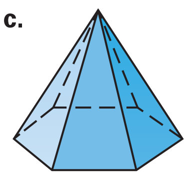
a.



b.



c.



SOLUTION

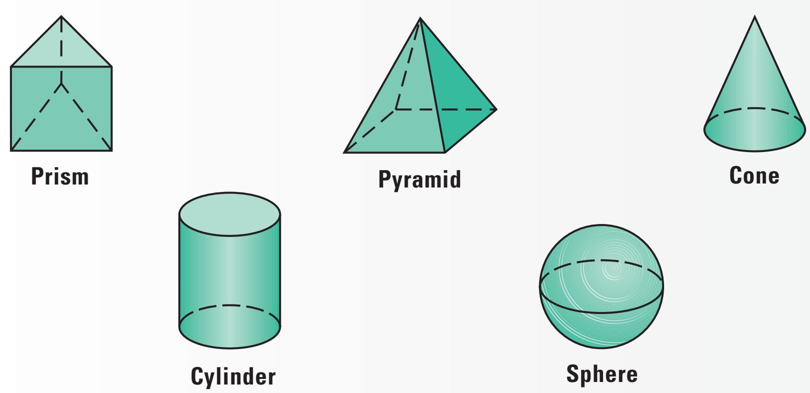
a. This is a polyhedron. It has 5 faces, 6 vertices, and 9 edges.

b. This is not a polyhedron. Some of its faces are not polygons.

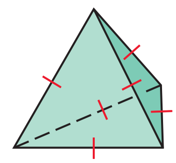
c. This is a polyhedron. It has 7 faces, 7 vertices, and 12 edges.

CONCEPT SUMMARY — TYPES OF SOLIDS

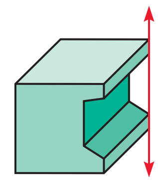
Of the five solids below, the prism and pyramid are polyhedra. The cone, cylinder, and sphere are not polyhedra.



A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is *nonconvex*, or *concave*.



regular, convex

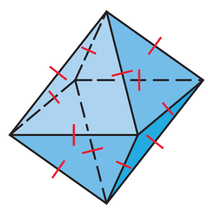


nonregular, nonconvex

EXAMPLE 2 *Classifying Polyhedra*

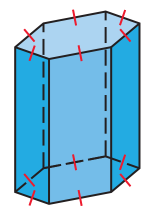
Is the octahedron convex? Is it regular?

a.



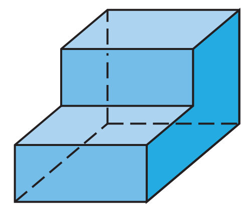
convex, regular

b.



convex, nonregular

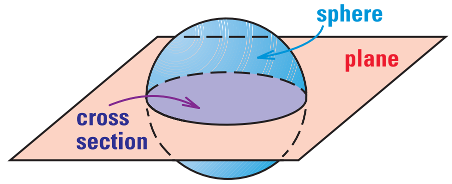
c.



nonconvex, nonregular

. . . .

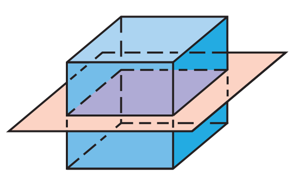
Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For instance, the diagram shows that the intersection of a plane and a sphere is a circle.



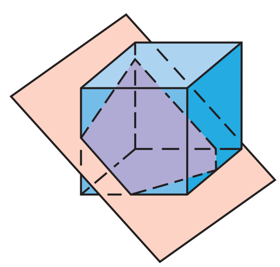
EXAMPLE 3 *Describing Cross Sections*

Describe the shape formed by the intersection of the plane and the cube.

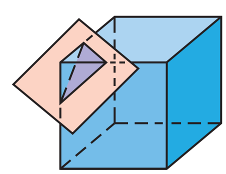
a.



b.



c.



STUDENT HELP

Study Tip
When sketching a cross section of a polyhedron, first sketch the solid. Then, locate the vertices of the cross section and draw the sides of the polygon.

SOLUTION

a. This cross section is a square.

b. This cross section is a pentagon.

c. This cross section is a triangle.

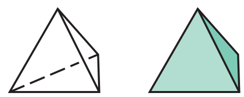
. . . .

The square, pentagon, and triangle cross sections of a cube are described in Example 3. Some other cross sections are the rectangle, trapezoid, and hexagon.

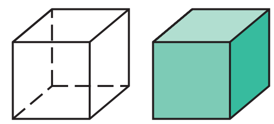
GOAL 2 Using Euler's Theorem

> **Student Help**
>
> **Study Tip**
> Notice that four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." A cube is sometimes called a *hexahedron*.

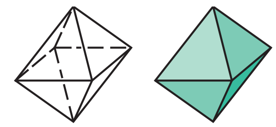
There are five regular polyhedra, called *Platonic solids*, after the Greek mathematician and philosopher Plato. The **Platonic solids** are a regular **tetrahedron** (4 faces), a cube (6 faces), a regular **octahedron** (8 faces), a regular **dodecahedron** (12 faces), and a regular **icosahedron** (20 faces).



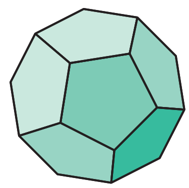
Regular tetrahedron
4 faces, 4 vertices, 6 edges



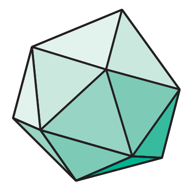
Cube
6 faces, 8 vertices, 12 edges



Regular octahedron
8 faces, 6 vertices, 12 edges



Regular dodecahedron
12 faces, 20 vertices, 30 edges



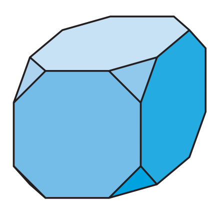
Regular icosahedron
20 faces, 12 vertices, 30 edges

Notice that the sum of the number of faces and vertices is two more than the number of edges in the solids above. This result was proved by the Swiss mathematician Leonhard Euler (1707–1783).

> **THEOREM**
>
> **THEOREM 12.1** ***Euler's Theorem***
>
> The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

EXAMPLE 4 *Using Euler's Theorem*

The solid has 14 faces; 8 triangles and 6 octagons. How many vertices does the solid have?



Solution

On their own, 8 triangles and 6 octagons have $8(3) + 6(8)$, or 72 edges. In the solid, each side is shared by exactly two polygons. So, the number of edges is one half of 72, or 36. Use Euler's Theorem to find the number of vertices.

$F + V = E + 2$ — **Write Euler's Theorem.**

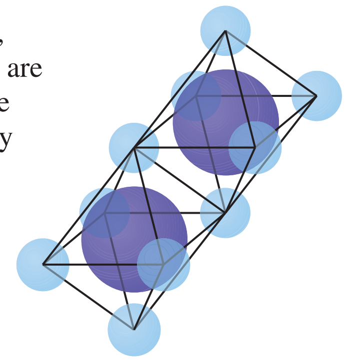
$14 + V = 36 + 2$ — **Substitute.**

$V = 24$ — **Solve for *V*.**

▶ The solid has 24 vertices.

EXAMPLE 5 *Finding the Number of Edges*

CHEMISTRY In molecules of sodium chloride, commonly known as table salt, chloride atoms are arranged like the vertices of regular octahedrons. In the crystal structure, the molecules share edges. How many sodium chloride molecules share the edges of one sodium chloride molecule?



SOLUTION

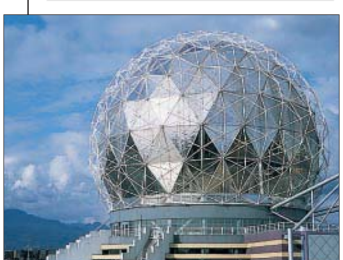
To find the number of molecules that share edges with a given molecule, you need to know the number of edges of the molecule.

You know that the molecules are shaped like regular octahedrons. So, they each have 8 faces and 6 vertices. You can use Euler's Theorem to find the number of edges, as shown below.

$F + V = E + 2$	**Write Euler's Theorem.**
$8 + 6 = E + 2$	**Substitute.**
$12 = E$	**Simplify.**

▶ So, 12 other molecules share the edges of the given molecule.

FOCUS ON APPLICATIONS



GEODESIC DOME The dome has the same underlying structure as a soccer ball, but the faces are subdivided into triangles.

APPLICATION LINK
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EXAMPLE 6 *Finding the Number of Vertices*

SPORTS A soccer ball resembles a polyhedron with 32 faces; 20 are regular hexagons and 12 are regular pentagons. How many vertices does this polyhedron have?

SOLUTION

Each of the 20 hexagons has 6 sides and each of the 12 pentagons has 5 sides. Each edge of the soccer ball is shared by two polygons. Thus, the total number of edges is as follows:

$E = \frac{1}{2}(6 \cdot 20 + 5 \cdot 12)$	**Expression for number of edges**
$= \frac{1}{2}(180)$	**Simplify inside parentheses.**
$= 90$	**Multiply.**

Knowing the number of edges, 90, and the number of faces, 32, you can apply Euler's Theorem to determine the number of vertices.

$F + V = E + 2$	**Write Euler's Theorem.**
$32 + V = 90 + 2$	**Substitute.**
$V = 60$	**Simplify.**

▶ So, the polyhedron has 60 vertices.

Guided Practice

Vocabulary Check ✓

1. Define *polyhedron* in your own words.

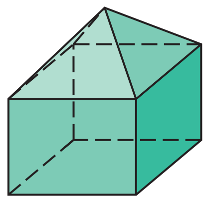
Concept Check ✓

2. Is a regular octahedron convex? Are all the Platonic solids convex? Explain.

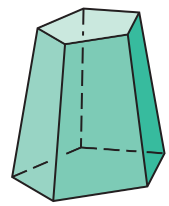
Skill Check ✓

Decide whether the solid is a polyhedron. Explain.

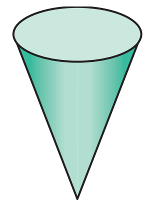
3.



4.



5.



Use Euler's Theorem to find the unknown number.

6. Faces: ___?___
Vertices: 6
Edges: 12

7. Faces: 5
Vertices: ___?___
Edges: 9

8. Faces: ___?___
Vertices: 10
Edges: 15

9. Faces: 20
Vertices: 12
Edges: ___?___

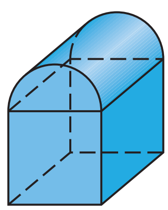
Practice and Applications

Student Help

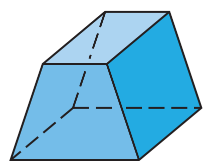
Extra Practice
to help you master skills is on p. 825.

Identifying Polyhedra **Tell whether the solid is a polyhedron. Explain your reasoning.**

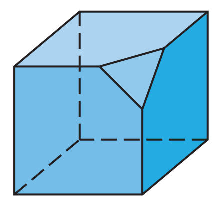
10.



11.

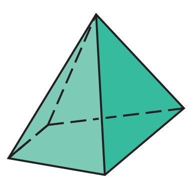


12.

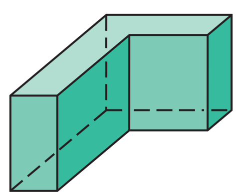


Analyzing Solids **Count the number of faces, vertices, and edges of the polyhedron.**

13.



14.



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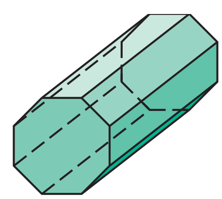
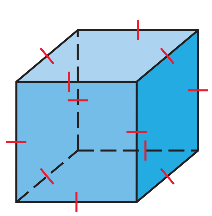


Student Help

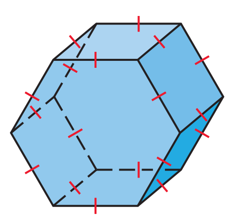
Homework Help
Example 1: Exs. 10–15
Example 2: Exs. 16–24
Example 3: Exs. 25–35
Example 4: Exs. 36–52
Example 5: Ex. 53
Example 6: Exs. 47–52

Analyzing Polyhedra **Decide whether the polyhedron is regular and/or convex. Explain.**

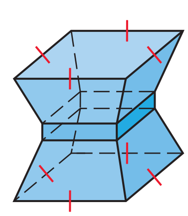
16.



17.



18.



LOGICAL REASONING **Determine whether the statement is *true* or *false*. Explain your reasoning.**

19. Every convex polyhedron is regular.

20. A polyhedron can have exactly 3 faces.

21. A cube is a regular polyhedron.

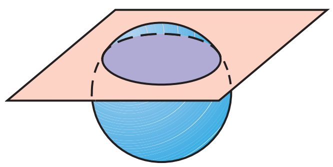
22. A polyhedron can have exactly 4 faces.

23. A cone is a regular polyhedron.

24. A polyhedron can have exactly 5 faces.

CROSS SECTIONS **Describe the cross section.**

25.



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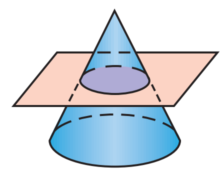
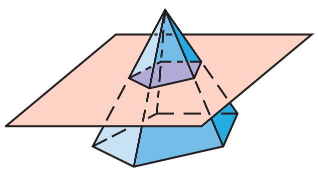
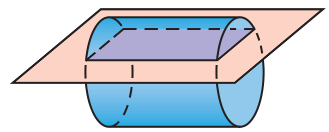


27.

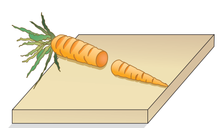


28.

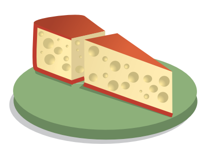


COOKING **Describe the shape that is formed by the cut made in the food shown.**

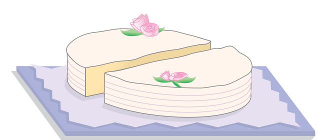
29. Carrot



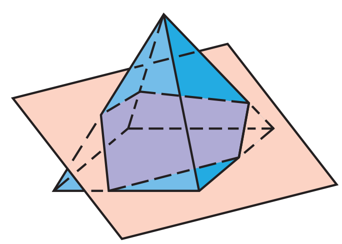
30. Cheese



31. Cake



CRITICAL THINKING **In the diagram, the bottom face of the pyramid is a square.**



32. Name the cross section shown.

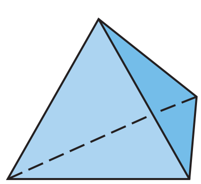
33. Can a plane intersect the pyramid at a point? If so, sketch the intersection.

34. Describe the cross section when the pyramid is sliced by a plane parallel to its bottom face.

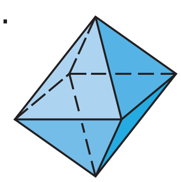
35. Is it possible to have an isosceles trapezoid as a cross section of this pyramid? If so, draw the cross section.

POLYHEDRONS **Name the regular polyhedron.**

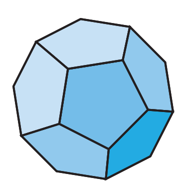
36.



37.



38.



FOCUS ON CAREERS

REAL LIFE **MINERALOGY**
By studying the arrangement of atoms in a crystal, mineralogists are able to determine the chemical and physical properties of the crystal.

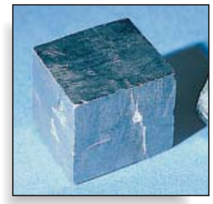
INTERNET CAREER LINK
www.mcdougallittell.com

CRYSTALS **In Exercises 39–41, name the Platonic solid that the crystal resembles.**

39. Cobaltite

40. Fluorite

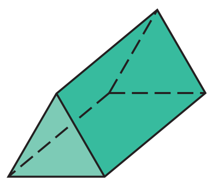
41. Pyrite



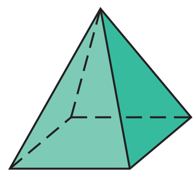
42. **VISUAL THINKING** Sketch a cube and describe the figure that results from connecting the centers of adjoining faces.

EULER'S THEOREM **In Exercises 43–45, find the number of faces, edges, and vertices of the polyhedron and use them to verify Euler's Theorem.**

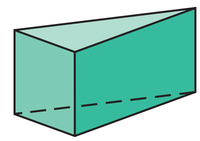
43.



44.



45.



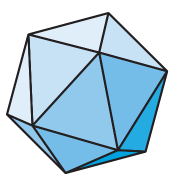
46. **MAKING A TABLE** Make a table of the number of faces, vertices, and edges for the Platonic solids. Use it to show Euler's Theorem is true for each solid.

STUDENT HELP

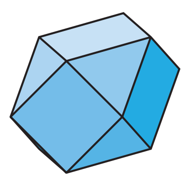
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 47–52.

USING EULER'S THEOREM **In Exercises 47–52, calculate the number of vertices of the solid using the given information.**

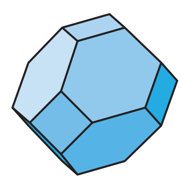
47. 20 faces;
all triangles



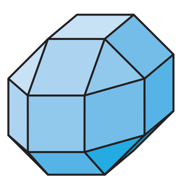
48. 14 faces;
8 triangles and
6 squares



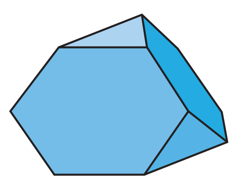
49. 14 faces;
8 hexagons and
6 squares



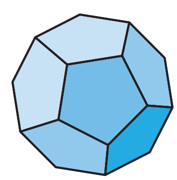
50. 26 faces; 18 squares and 8 triangles



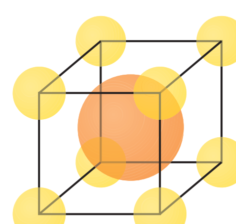
51. 8 faces; 4 hexagons and 4 triangles



52. 12 faces;
all pentagons



53. **SCIENCE CONNECTION** In molecules of cesium chloride, chloride atoms are arranged like the vertices of cubes. In its crystal structure, the molecules share faces to form an array of cubes. How many cesium chloride molecules share the faces of a given cesium chloride molecule?

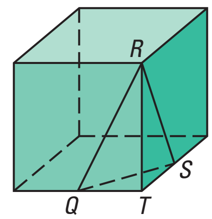


Test Preparation

54. MULTIPLE CHOICE A polyhedron has 18 edges and 12 vertices. How many faces does it have?

Ⓐ 4 Ⓑ 6 Ⓒ 8 Ⓓ 10 Ⓔ 12

55. MULTIPLE CHOICE In the diagram, Q and S are the midpoints of two edges of the cube. What is the length of $\overline{QS}$, if each edge of the cube has length h?



Ⓐ $\frac{h}{2}$ Ⓑ $\frac{h}{\sqrt{2}}$ Ⓒ $\frac{2h}{\sqrt{2}}$

Ⓓ $\sqrt{2}h$ Ⓔ $2h$

★ Challenge

SKETCHING CROSS SECTIONS **Sketch the intersection of a cube and a plane so that the given shape is formed.**

EXTRA CHALLENGE
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56. An equilateral triangle

57. A regular hexagon

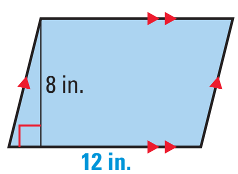
58. An isosceles trapezoid

59. A rectangle

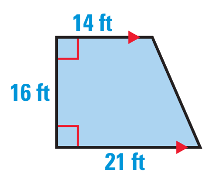
MIXED REVIEW

FINDING AREA OF QUADRILATERALS **Find the area of the figure.** (Review 6.7 for 12.2)

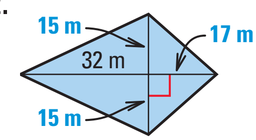
60.



61.



62.



FINDING AREA OF REGULAR POLYGONS **Find the area of the regular polygon described. Round your answer to two decimal places.** (Review 11.2 for 12.2)

63. An equilateral triangle with a perimeter of 48 meters and an apothem of 4.6 meters.

64. A regular octagon with a perimeter of 28 feet and an apothem of 4.22 feet.

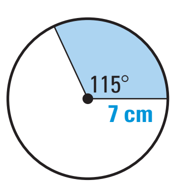
65. An equilateral triangle whose sides measure 8 centimeters.

66. A regular hexagon whose sides measure 4 feet.

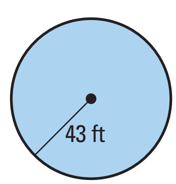
67. A regular dodecagon whose sides measure 16 inches.

FINDING AREA **Find the area of the shaded region. Round your answer to two decimal places.** (Review 11.5)

68.



69.



70.